Table 2 Normalized failure load for different stitching configurations

	No stitch[a]	Stitch at position a	Stitch at position b	Stitch at position c
Group 1, one stitch on stiffener	0.7741 ± 0.0326	0.8162 ± 0.0454	0.7851 ± 0.0713	0.7922 ± 0.0689
Group 2, one stitch on flange	0.7741 ± 0.0326	0.7994 ± 0.0573	0.7791 ± 0.0474	0.7275 ± 0.0822
	No stitch	One stitch	Two stitches	Three stitches
Group 3, on stiffener	0.7741 ± 0.0326	0.8162 ± 0.0454	0.8234 ± 0.0352	0.7227 ± 0.0320
Group 4, on flange	0.7741 ± 0.0326	0.7994 ± 0.0573	0.8162 ± 0.0460	0.7182 ± 0.0341

[a]All units in kN/cm.

Table 3 Measured data for moderate velocity ballistic impact

	Stitching configurations		
	1	2	3
Impacting velocity, m/s	102.3	103.1	103.1
Kinetic energy, J	188.90	191.83	191.83
Normalized damage zone, cm^2/J	0.87	0.485	0.665
Depth of damage zone, mm	0.65	1.45	1.9

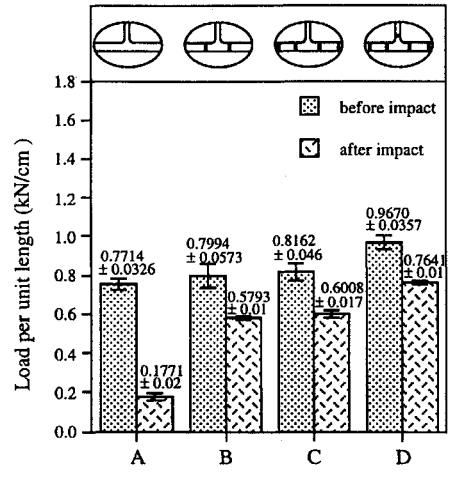

Fig. 3 Normalized failure pull-off load of the stiffened panels before and after impact.

specimens 2 and 3, the stitches of specimen 2 were closer to the stiffener, resulting in less damage area.

Conclusions

For stiffened panels, stitches on both the stiffener and the flange could increase the failure pull-off load by about 25%. As the skin side on the stiffener is subjected to impact, fiber stitches on the stiffener and flange can effectively reduce the crack growth and retain about 70–80% of the original strength of stiffener-to-skin joint.

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Analytical Model Updating and Model Reduction

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I. Introduction

In current vibration practice, structural dynamic characteristics are obtained by using either analytical approaches such as finite element modeling or experimental techniques such as modal testing. However, experience has shown that large discrepancies often exist between analytical predictions and experimental measurements. Under such circumstances, it is generally necessary to correct analytical models in the light of measured data, a process known as model updating.

Various methods to update analytical models using vibration test data have been developed.[1–8] These methods can, in general, be categorized as the reduced-order analytical model approach[1–4] and the full analytical model approach.[5–8] In the former, when the number of measured coordinates is less than the number of degrees of freedom (DOFs) specified in the analytical model, which is usually a case in practice, the analytical model to be updated is usually reduced via model reduction techniques such as the well-known Guyan reduction.[9] In the later, the analytical model is updated in its original full form by either using incomplete vibration test data directly[5–7] or expanding/interpolating unmeasured coordinates based on measured ones.[8] Such an approach is defined here as full analytical model approach, though in practice it is not possible to establish a full model since practical structures possess infinite number of DOFs.[10]

This paper examines the possibilities and limitations of current model updating practice so that further concerted research effort can be made towards more productive direction. It starts with investigating the effect of model reduction on the model updating process and hence demonstrating the practical difficulties associated with reduced-order analytical model updating approach. The advantages and possibilities of full analytical model updating are then discussed and the performance of some recently developed methods examined. Numerical results are given to illustrate the arguments developed.

II. Incompatibility Between Analytical and Experimental Models

In early development of model updating practice, it was always assumed that the number of DOFs specified in an analytical model is

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the same as that measured. In practice, however, there always exists incompatibility between an analytical model and measured data in terms of the coordinates employed. Usually, an analytical model employs a far greater number of coordinates than is practical for measured data. This is because 1) vibration testing is too expensive to permit measurements to be taken at many coordinates and 2) some coordinates may be either technically difficult to measure such as rotational DOFs or physically inaccessible such as internal DOFs. In practical modal tests of typical structures, the number of measured coordinates may be limited to, at most, a few hundreds, whereas the corresponding analytical models can be very detailed and may very often employ thousands of coordinates.

To tackle the problem of model updating in the presence of model incompatibility, such as the case of a plane frame structure shown in Fig. 1, in which 210 coordinates (70 nodes with 3 DOFs at each node) are employed in analytical modeling, whereas only 40 translational coordinates (20 hatched nodes with their translatioal DOFs) are measured, the most obvious way is perhaps to eliminate from the analytical model those coordinates that are not included in the measured data. This has led to the development of updating methods based on reduced-order analytical model approach that condense the analytical models using model reduction techniques and then update the reduced models.[1–4] Another way of tackling the problem is to directly correct the original full analytical model using measured incomplete data. The latter has prompted the development of methods based on full analytical model approach.[5–8] Both approaches are widely used nowadays in practice, and it is important to discuss what they can and cannot do when practical measurements are concerned.

III. Reduced-Order Analytical Model Approach

To resolve the problem of incompatibility, analytical models need to be reduced using reduction techniques such as the Guyan reduction, which is statically exact.[9] Let the original analytical mass and stiffness matrices be partitioned as

$$[M_a] = \begin{bmatrix} [M_a]_{11} & [M_a]_{12} \\ [M_a]_{21} & [M_a]_{22} \end{bmatrix} \qquad [K_a] = \begin{bmatrix} [K_a]_{11} & [K_a]_{12} \\ [K_a]_{21} & [K_a]_{22} \end{bmatrix} \quad (1)$$

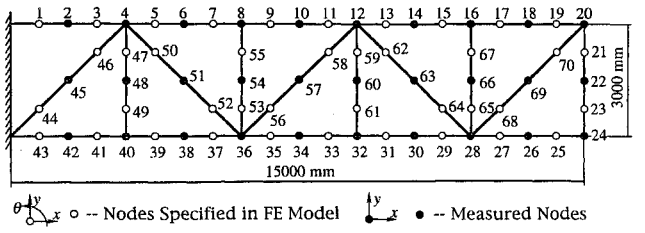

Fig. 1 Plane frame structure used in numerical simulation.

Then the Guyan reduction process reduces the analytical mass and stiffness matrices that are compatible with measured data as

$$[M_a]_R = [M_a]_{11} - [K_a]_{12}[K_a]_{22}^{-1}[M_a]_{21} - [M_a]_{12}[K_a]_{22}^{-1}[K_a]_{21} + [K_a]_{12}[K_a]_{22}^{-1}[M_a]_{22}[K_a]_{22}^{-1}[K_a]_{21} \quad (2)$$

$$[K_a]_R = [K_a]_{11} - [K_a]_{12}[K_a]_{22}^{-1}[K_a]_{21} \quad (3)$$

Though the eigenproperties of lower modes are closely preserved, localized modeling errors in the original analytical model will generally be scattered during such reduction. For example, when the cross-sectional area of the element between nodes 35 and 36 is overmodeled by 2 times, the stiffness modeling errors in the original model are localized within the 6 coordinates associated with these nodes. When the model is reduced to the measured coordinates (Fig. 1) using Guyan reduction, however, the localized modeling errors spread over almost all of the 40 measured coordinates as shown in Fig. 2. As a result, to update the reduced model, each individual element in the mass and stiffness matrices will have to be corrected, whereas only one variable is required to update the original model in this case. When all of the elements are to be updated, the number of measured modes available should be greater than the order n of the reduced model since for each mode one has

$$(\lambda_x)_i[\Delta M]_R\{\phi_x\}_i + [\Delta K]_R\{\phi_x\}_i = -(\lambda_x)_i[M_a]_R\{\phi_x\}_i - [K_a]_R\{\phi_x\}_i \quad (4)$$

$$\{\phi_x\}_i^T[\Delta M]_R\{\phi_x\}_i = 1 - \{\phi_x\}_i^T[M_a]_R\{\phi_x\}_i \quad (5)$$

which provide $n + 1$ equations in terms of the unknowns in the error mass and stiffness matrices and so n modes bring a total of $n \times (n+1)$ equations. The total number of unknowns is $n \times (n+1)$ since $[\Delta M]_R$ and $[\Delta K]_R$ are symmetric. This implies that at least n modes are required for the problem to become deterministic. Since n is generally much larger to model a test structure reasonably well, such demand on measured modes imposes practical difficulties, and even so the solution obtained will not be exact because of the errors involved in reduction process. These demonstrate the difficulty and limitation of the reduced-order analytical model approach. Also, it is worth mentioning that the applications of the updated reduced model are quite limited because of the scattering of localized modifications, and it perhaps cannot be directly used to perform important tasks such as structural modification and sensitivity analyses.

IV. Full Analytical Model Approach

Having recognized the practical difficulties and limitations associated with methods based on reduced-order analytical model, considerable efforts have been directed in recent years towards updating full analytical models using limited measured data.[5–8] The philosophy behind this practice is that although the order of analytical

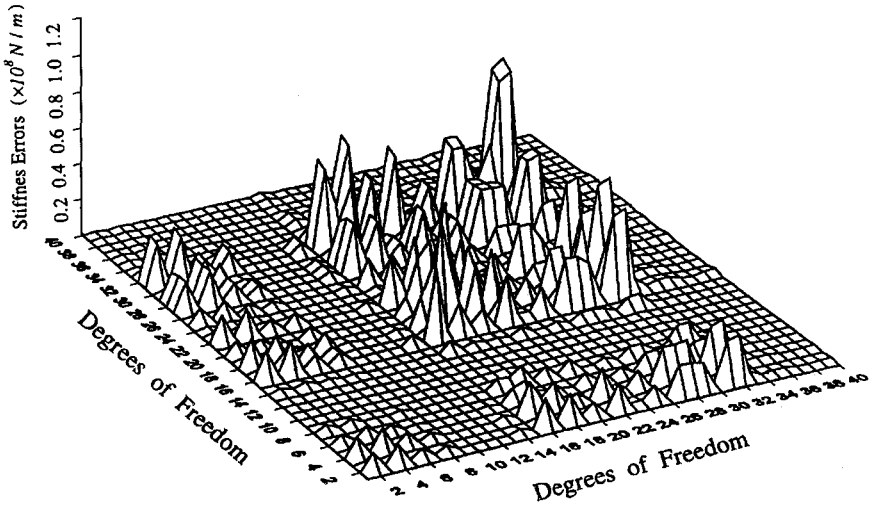

Fig. 2 Reduced-order stiffness modeling error matrix.

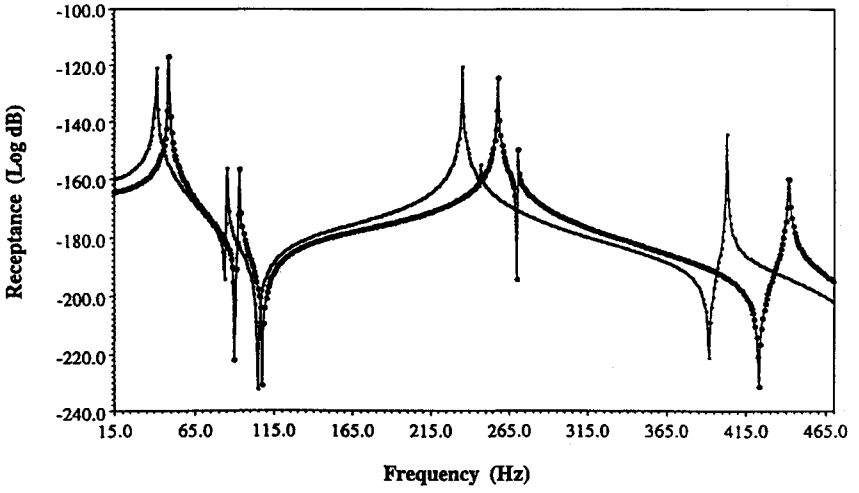

Fig. 3 Model updating using frequency response function method: -·-·-, analytical; ——, updated; and · · ·, experimental.

model is, in general, much larger than the number of measured coordinates, the number of variables to be updated is usually limited when structural connectivity is employed. The limited vibration test data available can still offer the possibility of updating the full model. Along this line of thinking, an effective method was developed to update full analytical models by expanding measured incomplete mode shapes based on interpolation.[8] By directly using incomplete measured data to avoid numerical errors due to interpolation, the inverse eigensensitivity method was proposed.[5] The method was later improved by removing the restriction of small magnitude of modeling errors and accelerating the speed of convergence so that it becomes generally applicable.[6] Assuming the mass and stiffness modeling errors can be expressed as

$$[\Delta M] = \sum_{k=1}^{N_M} \alpha_k [M_e]^{(k)} \qquad [\Delta K] = \sum_{k=1}^{N_K} \beta_k [K_e]^{(k)} \tag{6}$$

where $[M_e]^{(k)}$ and $[K_e]^{(k)}$ are the kth element mass and stiffness matrices, then correction coefficients α_k and β_k can be solved using the improved inverse eigensensitivity method as

$$\begin{bmatrix} \frac{\partial\{\bar{\phi}\}_1}{\partial\alpha_1} & \cdots & \frac{\partial\{\bar{\phi}\}_1}{\partial\alpha_{N_M}} & \frac{\partial\{\bar{\phi}\}_1}{\partial\beta_1} & \cdots & \frac{\partial\{\bar{\phi}\}_1}{\partial\beta_{N_K}} \\ \frac{\partial\bar{\lambda}_1}{\partial\alpha_1} & \cdots & \frac{\partial\bar{\lambda}_1}{\partial\alpha_{N_M}} & \frac{\partial\bar{\lambda}_1}{\partial\beta_1} & \cdot & \frac{\partial\bar{\lambda}_1}{\partial\beta_{N_K}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial\{\bar{\phi}\}_m}{\partial\alpha_1} & \cdots & \frac{\partial\{\bar{\phi}\}_m}{\partial\alpha_{N_M}} & \frac{\partial\{\bar{\phi}\}_m}{\partial\beta_1} & \cdots & \frac{\partial\{\bar{\phi}\}_m}{\partial\beta_{N_K}} \\ \frac{\partial\bar{\lambda}_m}{\partial\alpha_1} & \cdots & \frac{\partial\bar{\lambda}_m}{\partial\alpha_{N_M}} & \frac{\partial\bar{\lambda}_m}{\partial\beta_1} & \cdots & \frac{\partial\bar{\lambda}_m}{\partial\beta_{N_K}} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_{N_M} \\ \beta_1 \\ \vdots \\ \beta_{N_K} \end{Bmatrix}$$

$$= \begin{Bmatrix} \{\bar{\phi}_x\}_1 - \{\bar{\phi}_a\}_1 \\ (\lambda_x)_1 - (\lambda_a)_1 \\ \vdots \\ \{\bar{\phi}_x\}_m - \{\bar{\phi}_a\}_m \\ (\lambda_x)_m - (\lambda_a)_m \end{Bmatrix} \tag{7}$$

where $\partial\bar{\lambda}_r/\partial p$ and $\partial\{\bar{\phi}\}_r/\partial p$ are the improved eigenvalue and eigenvector sensitivities.[6]

The advantages of directly using receptance data in model updating has recently been discovered. Measured receptance data have the distinct advantage of automatically including the effect of all of the modes, not just the ones with natural frequencies in the range of interest. Further, they are more accurate in terms of data quality (no modal analysis errors) and plentiful in quantity as compared with their modal counterparts. Such advantages are employed in the recently developed updating method called the response function method.[7] Similarly, by expressing the modeling errors as Eq. (6), the correction coefficients can be determined as

$$\begin{bmatrix} \frac{\partial\{\bar{\alpha}(\omega_1)\}_i}{\partial\alpha_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_1)\}_i}{\partial\alpha_{N_M}} & \frac{\partial\{\bar{\alpha}(\omega_1)\}_i}{\partial\beta_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_1)\}_i}{\partial\beta_{N_K}} \\ \frac{\partial\{\bar{\alpha}(\omega_2)\}_i}{\partial\alpha_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_2)\}_i}{\partial\alpha_{N_M}} & \frac{\partial\{\bar{\alpha}(\omega_2)\}_i}{\partial\beta_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_2)\}_i}{\partial\beta_{N_K}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial\{\bar{\alpha}(\omega_m)\}_i}{\partial\alpha_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_m)\}_i}{\partial\alpha_{N_M}} & \frac{\partial\{\bar{\alpha}(\omega_m)\}_i}{\partial\beta_1} & \cdots & \frac{\partial\{\bar{\alpha}(\omega_m)\}_i}{\partial\beta_{N_K}} \end{bmatrix}$$

$$\times \begin{Bmatrix} \alpha_1 \\ \vdots \\ \alpha_{N_M} \\ \beta_1 \\ \vdots \\ \beta_{N_K} \end{Bmatrix} = \begin{Bmatrix} \{\tilde{\alpha}_x(\omega_1)\}_i - \{\tilde{\alpha}_a(\omega_1)\}_i \\ \{\tilde{\alpha}_x(\omega_2)\}_i - \{\tilde{\alpha}_a(\omega_2)\}_i \\ \vdots \\ \{\tilde{\alpha}_x(\omega_m)\}_i - \{\tilde{\alpha}_a(\omega_m)\}_i \end{Bmatrix} \tag{8}$$

where $\partial\{\bar{\alpha}(\omega)\}/\partial p$ are the generalized receptance sensitivity coefficients.[7]

These updating methods based on the full analytical model approach have been extensively applied to updating practices using simulated as well as real vibration test data and have been proven to be practically very useful. To illustrate, consider the example problem shown in Fig. 1 in which modeling errors were introduced by overestimating the cross-sectional areas of some of the elements. Only 40 (out of 210) coordinates were assumed to have been measured in a frequency range covering the first 5 modes. The response function method was then applied, and the updated model became exact after few iterations as shown in Fig. 3. Further, the thus updated model retains the necessary structural connectivities that are important in modification and sensitivity analyses.

V. Concluding Remarks

In practice, incompatibility between an analytical model and vibration test data always exists while the model needs to be updated for its intended applications. Two approaches are currently employed to address the problem: condensing the analytical model to those coordinates that are measured and updating the full analytical model using incomplete measured data. The former involves a process of reduction that, unfortunately, spreads localized modeling errors, leading to a dramatic increase in the number of coefficients to be corrected. In fact, it has been established that at least n (order of the reduced model) modes need to be measured before deterministic results are obtained. Further, the practical usefulness of the thus updated model is also quite limited. The latter, on the other hand, preserves all of the details that modern analytical modeling techniques can offer and corrects the analytical model using incomplete test data so that it better describes the actual structural behavior. It

is therefore the author's belief that future research effort should be more focused on full analytical model updating and, in particular, more emphasis should be placed on the direct application of measured receptance data rather than modal data. The discussions made throughout are illustrated by a realistic example problem.

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Stochastic Optimization Using the Stochastic Preconditioned Conjugate Gradient Method

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Introduction

THIS Note describes the effectiveness of the stochastic preconditioned conjugate gradient (SPCG) solver developed for solving large systems of equations arising from repeated response calculations, where values of the input variables (structure properties and loading) are perturbed about a central point.[1] We extend this earlier work and examine the speedups achievable with this solver over direct solvers for probabilistic mechanics and optimization problems. The key to the efficiency of the SPCG solver is a preconditioning strategy that capitalizes on the repeated solutions required for probabilistic mechanics and optimization.

In this work the SPCG solver has been integrated into NIKE3D and used to conduct a Monte Carlo simulation (MCS) and a stochastic shape optimization of a cantilever beam. The MCS test case is an application (similar to sensitivity analysis) where the repeated response calculations can be small or large perturbations of the structure. In contrast, the optimization test case employs fitted response surfaces that are obtained by analyzing relatively large perturbations of the structure, where the SPCG solver is used to reduce the cost of this process. For these examples, the SPCG solver is shown to outperform a Jacobi preconditioned conjugate gradient (PCG) solver by an order of magnitude and a direct solver by more than a factor of 2.

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Methodology

The SPCG solver is identical to the widely used PCG method[2] except that it employs a unique preconditioning strategy that capitalizes on the repeated solutions required for probabilistic mechanics and optimization. The SPCG method is defined as follows (see Fig. 1). First, the preconditioning matrix $\boldsymbol{K}_0$ is formed by assembling the stiffness matrix obtained from the center point of the perturbation sequence. This is illustrated in Fig. 1 for a response surface generation example, where $\boldsymbol{K}_0$ is evaluated using the central values of design variables x and y. [When the perturbations are based on samples of the problem random variables (e.g., conventional MCS techniques) or finite differences of design or random variables (e.g., during optimization or FORM/SORM methods), then $\boldsymbol{K}_0$ may be obtained using the mean values of the random variables or the current estimates of the design variables.] Next, this matrix is factored using Cholesky decomposition. Then for each perturbation of the structure, the following steps are taken. The stiffness matrix $\boldsymbol{K}_i$ and the load vector $\boldsymbol{f}_i$ for the ith perturbation are assembled. The initial guess for the unknown displacements $\boldsymbol{z}_i^0$ is computed by solving $\boldsymbol{K}_0\boldsymbol{z}_i^0 = \boldsymbol{f}_i$ using the factored matrix, where $\boldsymbol{f}_i$ is the current simulation history realization of the load or the loads corresponding to the current perturbation (the mean or center point load vector $\boldsymbol{f}_0$ can also be used). Lastly, the resulting linear system of equations is solved using a PCG algorithm, where $\boldsymbol{K}_0^{-1}$ is used as the preconditioner and $\boldsymbol{z}_i^0$ is used as the initial guess. Using this approach, the preconditioning matrix will approximate the stiffness matrix generated for individual perturbations and it need only be factored once before the commencement of the perturbation sequence.

In practice we have found the SPCG algorithm to be highly efficient even for large perturbations about the central point. Performance trends of the SPCG solver relative to a direct skyline solver[3] and the Jacobi PCG (JPCG, PCG with diagonal preconditioning) can be estimated by considering the number of floating point operations required for each solver. A summary of the operation counts for banded versions of each solver is given in Table 1, where one operation is defined as one multiplication (or division) followed by an addition.

By assuming that $n_b \gg 4$, the following ratio of operation counts may be derived for the SPCG and direct solver:

$$\frac{\text{SPCG}}{\text{Direct}} = \frac{1}{n_h} + \frac{8n_i + 12}{n_b}$$

Based on this ratio, the performance advantages of the SPCG solver relative to the direct solver increase for three-dimensional problems that have a large bandwidth and require a large number of repeated response calculations.

Numerical Implementation

Herein, an SPCG solver has been integrated into NIKE3D, a general-purpose nonlinear finite element code. For probabilistic analysis (e.g., MCS), it is designed to utilize a preconditioning matrix obtained from an initial mean-value stiffness matrix to minimize the computation time for the repeated structural analyses. It has also been designed to operate within the context of a multidisciplinary stochastic optimization (MSO) shell that has been developed for NASA Lewis Research Center.[4]

The SPCG solver is designed to operate as follows within the MSO shell. Given a sequence of analyses to be conducted for response surface generation, the initial center point analysis is solved using a direct solver. Using a compiler directive, the resulting factorized global stiffness matrix from this run is stored in shared memory (to avoid any file input/output) and then repeatedly accessed and used to precondition each subsequent NIKE3D analysis in the designed experiment series. For MCS, the implementation is similar. The preconditioning matrix is created by factorizing the stiffness matrix obtained using the mean values of the random variables.

Example Problems

Example 1: Monte Carlo Simulation of Cantilever Beam

An MCS was performed using the SPCG solver to evaluate the expected tip deflection of a three-dimensional cantilever beam ($8 \times 8 \times 8$ element mesh, three-dimensional trilinear elements, 1944